ABSTRACT
of dissertation for the Philosophy Doctor (PhD) degree in specialty
"6D060400 – Physics" by
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on the topic "**INVESTIGATION OF NONLINEAR SOLITON MODELS OF FERROMAGNETS**"

The dissertation deals with the theoretical investigation of some nonlinear models such as the Heisenberg model for ferromagnets, which describes the propagation of spin waves in a nonlinear medium.

Relevance of the dissertation theme.

Recently, the interest of scientists in the theoretical and experimental study of nonlinear phenomena in magnetic media has increased. This interest is mainly due to the fact that magnetic crystals are widely used in many fields where the nonlinear properties of magnets are exploited. For example, the speed of some components of modern computers depends on the dynamics of cylindrical magnetic domains, which are essentially nonlinear structures of ferromagnetic materials. Nonlinear excitation of magnetic crystals is also possible.

An important example of solitons in magnets, i.e., magnetic solitons, is a domain boundary separating uniformly magnetized domains with different magnetization directions. Of particular interest are cylindrical magnetic domains, which can be used to transmit and record information in modern computers.

Nonlinear phenomena can be described by the interaction of elementary excitations of systems of interacting spins – magnons. Magnonic interaction processes not only plays an important role in shaping the response of a magnet to an external field, but also significantly influences the kinetic and relaxation properties of magnetic systems. Until recently, the theoretical description of nonlinear phenomena in magnets used the concept of weak nonlinearity, which means that the interaction energy of magnons is small compared to the energy of a "free" magnon. However, in the case of magnets, contradictory situations may arise when the interaction energy of magnons turns out to be comparable to the energy of a "free" magnon. In such cases, the use of concepts of a weakly nonlinear description is no longer appropriate for the effect under investigation, and it is necessary to introduce new concepts and develop methods for describing strongly nonlinear phenomena in magnets.

In the last thirty years, active research on the nonlinear properties of spin waves in ferromagnets has been initiated. One of the areas of fundamental research that has played an important role in the theoretical study of other nonlinear wave processes is the theory of solitons. Solitons in magnets are waves with a magnetic moment localized in space and are considered theoretically as partial solutions of the Landau-Lifshitz, Heisenberg, and nonlinear Schrödinger equations satisfying certain boundary conditions. Currently, the theory of solitons is one of the rapidly developing areas of modern physics. These studies are of great interest in applications such as the development of nonlinear devices for processing and transmitting data in telecommunication systems, laser systems, nonlinear transmission lines, optical switches, and communication lines.

There are two types of solitons - static and dynamic. Dynamic solitons, i.e., envelope-wave solitons, can occur, for example, in magnetically ordered media and optical fibers. Ferromagnetic films and structures based on them are best suited for the study of solitons of envelope waves in microwave radiation. Spin waves can propagate in such media. The envelope soliton of spin waves can be described by the nonlinear Schrödinger equation (NLS). Some studies show that for a more accurate description of soliton processes, it is necessary to modify the classical NLS equation by adding terms describing linear and nonlinear damping, excitation, and higher dispersion terms. As a result, more and more researchers are developing new theoretical models that describe the soliton phenomena observed in experiments. The results presented in this paper are relevant.

The development of soliton models for ferromagnets was intensively pursued after scientists began to study the properties of spin wave propagation in nonlinear media. The main aspects in the study of the theory of solitons in magnets are due to the following reasons:

First, more and more researchers are turning their attention to the development of new theoretical models describing the soliton phenomena observed in experiments.

Second, since much work is being done in the framework of dissipation-free models, much more interesting questions arise (from a physics point of view): how do solitons change in magnetic dipole interactions?

Starting from the above mentioned modern problems of theoretical physics, the dissertation work: "**Investigation of nonlinear soliton models of ferromagnets**" is devoted to the theoretical study of some nonlinear models, such as the Heisenberg model of ferromagnets, which describes the propagation of spin waves in a nonlinear environment.

The main goal of work. Theoretical study of some nonlinear models of ferromagnets of the Heisenberg type and study of soliton solutions of the equations describing the interaction of spin waves.

To achieve the above goal, the following **tasks** must be accomplished:

1. Search for nonlinear evolution equations that are geometrical and gauge equivalent to the generalized ferromagnetic Heisenberg equation.

2. Investigating the differential geometry of equations that are generalized ferromagnetic Heisenberg models with self-consistent sources.

3. Obtaining soliton solutions of nonlinear Schrödinger-type equations and studying the dynamics of their change.

The object of the research. Nonlinear models of spin systems.

The subject of the study. Search for soliton solutions of the nonlinear Schrödinger equation, Heisenberg ferromagnet equation.

Research methods. To obtain soliton solutions describing the dynamics of nonlinear waves, the Darboux transformation method was used, which was actively employed to study the evolution of soliton dynamics.

Scientific novelty of the dissertation results

The novelty and originality of the dissertation work lies in the fact that it is the **first time**:

1. A geometric and gauge connection has been established between the generalized Heisenberg ferromagnet equation and the Konno-Oono equation. Soliton solutions of Konno-Oono are constructed.

2. The differential geometry of the generalized Heisenberg ferromagnet model with self-consistent sources and the generalized complex short pulse equation have been studied.

3. One- and two-soliton solutions of nonlinear Schrödinger-type equations describing the spontaneous interaction of optical pulses were constructed and their conservation laws were found.

The main provisions for the defense:

1. The generalized Heisenberg ferromagnet model with self-consistent sources in case of the identity of the spin vector $\mathbf{A}$ and the basis vector $\mathbf{e}_1$ $(\mathbf{A} \equiv \mathbf{e}_1)$ is geometrically and gauge equivalent to the equation, which is a generalized form of the complex short-pulse equation.

2. The generalized Heisenberg ferromagnet model with self-consistent sources in the absence of potential $(\mathbf{W} = 0)$ is a special case of the generalized Heisenberg ferromagnet model, which is geometrically and gauge equivalent to the complex coupled dispersionless Konno-Oono equation, and its solution corresponds to a soliton.

3. The two-dimensional nonlinear Schrödinger equation with “attraction” $(\delta = 1)$ which describes spin waves in ferromagnets, has one- and two-soliton solutions, and its two-soliton collisions are elastic, that is, they retain their shape and speed after interaction.

Practical and theoretical importance of the dissertation

The dissertation work is theoretical in nature. The results obtained can be used in the theory of solitons, for example, in the study of the dynamics of the nonlinear Heisenberg ferromagnet equations. The results of the dissertation can also be used in obtaining solitons and soliton-like solutions for work in the field of nonlinear medium, electromagnetic waves in optical transmission lines, etc.

Some of the results of the dissertation can be used in the framework of elective courses (lectures “Theory of Solitons” and “Theory of Hydrodynamic Systems”) in education.

Author's personal contribution. During the research process, the author participated in all steps of the work, performed all calculations under the guidance of a scientific supervisor, prepared graphics of the solutions found, and personally prepared the publications.

Reliability and validity of the results.

The reliability of the scientific results is confirmed by references of foreign scientists to publications in foreign journals with high impact factor, as well as by conference proceedings.

Approbation of the dissertation

The results obtained in the dissertation were presented and discussed at the conferences:

1. 5th International conference Modern problems of applied mathematics and information technology - Al-Khorezmiy (Bukhara, 2016. –November 9-10).
2. 3rd International Conference “Astrophysics, Gravity and Cosmology” (Astana 2016. –November 30 – December 2).
3. 22th international conference Geometry, Integrability and Quantization (Varna, 2019. – June 8-13).
4. The 26th international conference on Integrable Systems and Quantum symmetries (Prague, 2019. – July 8 - 12).

In addition, the obtained results were presented and discussed at scientific seminars of the Department of General and Theoretical Physics of L.N. Gumilyov ENU and at seminars of the Eurasian International Center for Theoretical Physics.

Publications on the topic of the dissertation

Based on the results of the dissertation, 12 articles were published in Kazakh and foreign journals, including 2 articles in a foreign journals with high impact factor listed in the databases Web of Science (quartile Q2, impact factor 1.783 and 3.1) and Scopus (Percentile 56 and 73, Physics and Astronomy category); 3 articles in journals recommended by CQASE MSHE RK; 7 articles in proceedings of conferences indexed in Web of Science and Scopus databases. Key publications:

1. Zhassybayeva M., Bekova G., Yesmakhanova K., Myrzakulov R. Integrable motions of curves of the induced Fokas–Lenells equation // Optik. -2023. –Vol. 286. – P.170979. Impact Factor 2022 – 3.1. Q2, Percentile 73.
2. Yesmakhanova K., Nugmanova G., Shaikhova G., Bekova G., Myrzakulov R. Coupled Dispersionless and Generalized Heisenberg Ferromagnet Equations with Self-Consistent Sources: Geometry and Equivalence // International Journal Geometrical Methods in Modern Physics. – 2020. – Vol. 17, No 7. – P. 2050104. Impact Factor 2019 – 1.287. Q2, Percentile 47.
3. Yesmahanova, K.R., Shaikhova, G.N., Bekova, G.T., Myrzakulova, Z.R. Determinant reprentation of dardoux transformation for the (2+1)-dimensional Schrodinger-Maxwell-Bloch equation //Advances in Intelligent Systems and Computing. – 2016. – Vol. 441. – P. 183-198.
4. Yesmakhanova K., Shaikhova G., Bekova G. Soliton solutions of the Hirota’s system //AIP Conf. Proc. –2016. – Vol. 1759. –P. 020147. Процентиль 15.
5. Shaikhova G., Yesmakhanova K., Bekova G., Ybyraiymova S. Conservation laws of the Hirota-Maxwell-Bloch system and its reductions // Journal of Physics: Conference Series. – 2017.–Vol. 936, N1. –P. 012098. Percentile 22.
6. Bekova G., Shaikhova G., Yesmakhanova K. Travelling wave solutions for the two-dimensional Hirota system of equations // AIP Conference Proceedings. – 2018.–Vol. 1997, –P. 020039. Percentile 15.

7. Bekova G., Shaikhova G., Yesmakhanova K., Myrzakulov R. Conservation Laws for Two Dimensional Nonlinear Schrodinger Equation // AIP Conference Proceedings. –2019.–vol. 2159, –P.030003. Percentile 15.

8. Bekova G., Shaikhova G., Yesmakhanova K., Myrzakulov R. Darboux transformation and soliton solution for generalized Konno-Oono Equation // Journal of Physics: Conference Series. –2019. –vol.1416. –P. 012003. Percentile 22.

9. Ismailova F.Ye., Bekova G., Shaikhova G.N. Traveling wave solitions for the two-dimensional generalized nonlinear Schrodinger equation // Vestnik KazNRTU. – 2019. – No 4 (134). - pp. 534-540.

10. Бекова Г.Т., Мусатай С.С., Абыканова Б.Т. Решения типа разрушительных волн двумерного нелинейного уравнения Шредингера // Вестник КазНИТУ. -2020. №2 (138). стр.-715-721.

H-index and citation of works. Doctoral student Bekova G.T. has the next following scientometric indicators in the databases Web of Science h-index-7 (Citation 114) and Scopus h-index-9 (Citation 124).

Connection of the dissertation topic with the plans for the scientific work.

The work was carried out in accordance with the research plans for the following projects:

1) “Research of the generalized nonlinear Schrödinger equations and their integrable reductions”, (2015-2017) state registration No. 0115РК01346. Agreement No. 268 dated 02/04/2015. Supervisor K.R. Yesmakhanova.

2) “Modeling of wave processes in magnetics on the basis of the theory of solitons” (2022-2024), AP 14972426 Supervisor G.T. Bekova.

Structure and scope of the dissertation.

The dissertation consists of an introduction, 3 sections, a conclusion and a list of references containing 195 titles, includes 101 pages of typewritten text, including 12 figures.